MATRH260—Week 7 Lab **Name:** Lisa Frazier 11/10/17

Logarithmic Integrals

Examine each example below then answer questions 1–3.

① Integral: $\int \frac{1}{x} dx$ Solution: ln |x| $+C$	② Integral: $\int \frac{1}{x-3} dx$ Solution: $\ln\|x+3\|+C$
③ Integral: $\int \frac{2x}{x^2+4} dx$ Solution: $ln\|x^2+4\|+C$	④ Integral: $\int \frac{1}{2x-3} dx$ Solution: $\frac{1}{2} ln\|2x-3\|+C$

1) What is the formula for finding $\int \frac{du}{u}$? $= \ln|u| + C$

2) How is substitution being used to solve examples ③ and ④? Show u and du.

③ $u = x^2+4$ $\frac{du}{dx} = 2x$ $du = 2x\,dx$	④ $u = 2x-3$ $\frac{du}{dx} = 2$ $du = 2\,dx$

3) Why can't we find the integral $\int \frac{1}{x}\ dx$ using the following set-up? $\int \frac{1}{x}\ dx = \int x^{-1}\ dx$

$= \frac{x^{-1+1}}{-1+1}$

$= \frac{1}{0}$ (division by zero = undifined)

Derivatives of $\infty \neq \frac{1}{x}$

4) Fill in the blanks below with the correct information for the following integral: $\int \frac{3}{4x+5}\,dx$

u =	$4x-5$	du =	$4\,dx$
Substitution =	$\frac{3}{4}\int \frac{du}{u}$	Integration =	$\frac{3}{4}(\ln\lvert u\rvert)+C$
Final answer =	$\frac{3}{4}\ln\lvert 4x+5\rvert+C$		

5) Fill in the blanks below with the correct information for the following integral: $\int \frac{e^{3x^2}}{x^3-5}\,dx$

u =		du =	
Substitution =		Integration =	
Final answer =			

6) Fill in the blanks below with the correct information for the following integral: $\int \frac{6x\,e^{3x^2}}{e^{3x^2}-5}\,dx$

u =	$e^{3x^2}-5$	du =	$e^{3x^2}\cdot 6x\,dx$
Substitution =	$\int \frac{du}{u}$	Integration =	$\ln\lvert u\rvert+C$
Final answer =	$\ln\lvert e^{3x^2}-5\rvert+C$		

Exponential Integrals

Examine each example below and answer questions 7–10.

① Integral: $\int e^x dx$ Solution: $e^x + C$	② Integral: $\int 2xe^{x^2} dx$ Solution: $e^{x^2} + C$
③ Integral: $\int e^{3x} dx$ Solution: $\frac{1}{3}e^{3x} + C$	④ Integral: $\int sec^2(2x)e^{\tan(2x)} dx$ Solution: $\frac{1}{2}e^{\tan(2x)} + C$

7) What is the formula for finding $\int e^u du$? $e^u + C$

8) How is substitution being used to help solve examples ②, ③, and ④?
Show u, du, and the substituted integral for each. Indigration

②	③	④
$u = x^2$ $du = 2x dx$ $\int e^u du$	$u = 3x$ $du = 3 dx$ $\int \frac{e^u}{3} du$	$u = \tan(2x)$ $du = \sec^2(2x) \cdot 2dx$ $\int \frac{e^u}{2} du$

9) Fill in the blanks below with the correct information for the following integral: $\int 5x^2 e^{x^3}\, dx$

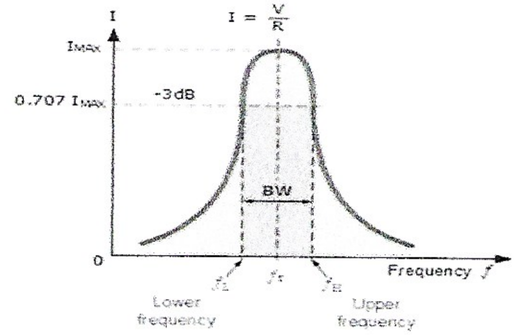
u =	x^3	du =	$3x^2$
Substitution =	$\int \frac{5}{3} e^{u} du$		
Integration =	$\frac{5}{3} e^{u} + C$		
Final answer =	$\frac{5}{3} e^{x^3} + C$		

10) Fill in the blanks below with the correct information: $\int \sec^2(8x)\, e^{3\tan(8x)}\, dx$

u =	$3\tan(8x)$	du =	$3\sec^2(8x)\cdot 8$
Substitution =	$\int \frac{1}{24} e^{u} du$		
Integration =	$\frac{1}{24} e^{u} + C$		
Final answer =	$\frac{1}{24} e^{3\tan(8x)} + C$		

Bandwidth of a Series Resonance Circuit

The area under the curve shows all acceptable signals with frequency greater than f_L and less than f_H that can pass through the resonance circuit. This is used in radio receivers to tune for different channels.



11) If the following equation I(f) represents the curve for a frequency bandwidth similar to the one above, find the area under the curve from f_L= .8 H to f_H. = 1.2 H. Round all values to the 100ths.

$$I(f) = \frac{1}{.2\sqrt{2\pi}} \cdot e^{\frac{-(f-1)^2}{2(.2)^2}}$$, xmin= -2, xmax = 3, ymin= -2, ymax = 3

Trigonometric Integrals

Some of the trig integrals are the reverse of their derivatives.

$\int \cos x dx$ = sin x + C	$\int \sin x$ dx = -cos x + C
$\int \sec^2 x dx$ = tan x + C	$\int \csc^2 x$ dx = -cot x + C
$\int \sec x \tan x dx$ = sec x + C	$\int \csc x \ \cot\ x\ dx$ = -csc x + C

And some require reciprocal identities and integrate to natural logs.

$\int \sec x dx$ = ln |sec x + tan x| + C	$\int \csc x$ dx = -ln |csc x + cot x| + C
$\int \cot x \ \ dx$ = ln|sin x| + C	$\int \tan x \ \ dx$ = -ln|cos x| + C

All should be memorized; remember that you can always check an integrand by differentiating.

Sine and Cosine

① Integral: $\int \sin(2x)\,dx$ Solution: $-\frac{1}{2}\cos(2x) + C$	② Integral: $\int \cos(4x)\,dx$ Solution: $\frac{1}{4}\sin(4x) + C$
③ Integral: $\int -x\ sin(x^2+5)dx$ Solution: $\frac{1}{}\cos(x^2+5) + C$	④ Integral: $\int 9x^2\cos(x^3-15)\,dx$ Solution: $\frac{1}{}\sin(x^3-15)$ + C

12) Use u-substitution and the above examples to find $\int e^{2x}\sin(e^{2x})\,dx$

Substitute $e^{2x} = u$

$2e^{2x}\,dx = du$

$\Rightarrow e^{2x}\,dx = \frac{du}{2}$

$= \frac{1}{2}\int \sin(u)\,du$

$= -\frac{1}{2}\cos(u) + C$

$= -\frac{1}{2}\cos(e^{2x}) + C$

13) Use u-substitution to find $\int 3x\, cos(x^2+8)\,dx$

Substitute $x^2+8 = u$

$\Rightarrow 2x\,dx = du$

$\Rightarrow x\,dx = \frac{du}{2}$

$= \int \frac{2}{3}\cos u\,du$

$= \frac{2}{3}\sin u + C$

$= \frac{3}{2}\sin(x^2+8) + C$

Secant and Cosecant, Tangent and Cotangent

Examine each example below and answer the following questions.

	Integral	Solution
i.	$\int \sec(6x)\,dx$	$\frac{1}{6} ln\lvert\sec(6x)+\tan(6x)\rvert + C$
ii.	$\int x^4 \csc(x^5-7)dx$	$-\frac{1}{5} ln\lvert csc(x^5-7)+cot(x^5-7)\rvert + C$
iii	$\int e^{\frac{1}{2}x} sec\left(e^{\frac{1}{2}x}\right) tan\left(e^{\frac{1}{2}x}\right) dx$	$2sec\left(e^{\frac{1}{2}x}\right) + C$
iv.	$\int \csc(10x)\cot(10x)\ dx$	$-\frac{1}{10}\csc(10x) + C$
v.	$\int \cot(10x)\ dx$	$\frac{1}{10} ln\lvert sin(10x)\rvert + C$
vi.	$\int 2x\tan(10x^2)\ dx$	$-\frac{1}{10} ln\ \lvert cos(10x^2)\rvert + C$

*Note: absolute value symbols ensure that the argument for the natural logarithm is always positive and the result is real.

14) Use differentiation to show that each of the solutions for the integrals i., iii., v., vi. above is true.

i) $y = \frac{1}{6} \ln \lvert \sec(6x) + \tan(6x) \rvert + C$

$\frac{dy}{dx} = \frac{1}{6}\left[\frac{1}{\sec(6x)\tan(6x)} \times (6\sec(6x)\tan(6x) + 6\sec^2(6x))\right]$

$= \sec(6x)$

iii) $y = 2\sec(e^{1/2x}) + c$

$\frac{dy}{dx} = 2\sec(e^{1/2x}) \cdot \tan(e^{1/2x}) \cdot e^{1/2x} \cdot \frac{1}{2}$

$= e^{1/2x}\sec(e^{1/2x})\tan(e^{1/2x})$

v) $y = \frac{1}{10} \ln \lvert \sin(10x) \rvert + C$

$\frac{du}{dx} = \frac{1}{10} \times \frac{1}{\sin(10x)} \times \cos(10x) \times 10$

$= \frac{\cos(10x)}{\sin(10x)}$

$= \cot(10x)$

vi) $y = -\frac{1}{10} \ln \lvert \cos(10x^2) \rvert + C$

$$\frac{dy}{dx} = \frac{-1}{\cancel{10}} \times \frac{1}{\cos(10x^2)} \times (-\sin(10x^2) \times \cancel{10} \cdot 2x$$

$$= 2x \frac{\sin(10x^2)}{\cos(10x^2)}$$

$$= 2x \tan(10x^2)$$

15) If $\int \cot(u)\, du = \ln|\sin u| + C$ what does $\int \cot\left(\frac{1}{2}x\right) dx$ =

Substitute $\frac{1}{2}x = u$

$\Rightarrow \frac{1}{2} dx = du$

$\Rightarrow dx = 2du$

$\therefore \int \cos u \cdot 2\, du = 2 \ln|\sin u| + C$

$= 2 \ln|\sin(\frac{1}{2}x) + C$

16) Graph the function sin(e^x) and find the area from 0 to $\frac{\pi}{4}$.
xmin = -1, x-max = 5, y min=-3, y-max = 2, radian mode.

17) $\int 3x^5 csc(x^6)dx$ Substitute $x^6 = u$

$\Rightarrow 6x^5\, dx = du$

$\Rightarrow 3x^5\, dx = \frac{du}{2}$

$\int csc(u)\, \frac{du}{2} = -\frac{1}{2} \ln|csc\, u + cot\, u| + C$

$= -\frac{1}{2} \ln|csc(x^6) + cot(x^6)| + C$

18) $\int e^{-3x}sec(e^{-3x})tan(e^{-3x})dx$ using Rule iii)

$= -\frac{1}{3} sec(e^{-3x}) + C$

19) Break the integral into three less complicated integrals, then integrate.
Hint: rewrite using the idea that $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$, reduce, rewrite using reciprocal identities, then integrate.

$$\int \frac{\sin^2(x)+\sin(x)+\cos(x)}{\mathrm{si}\ {}^{2(\ )}}\, dx$$

$$= \int \frac{\sin^2 x}{\sin^2 x} + \frac{\sin x}{\sin^2 x} + \frac{\cos x}{\sin^2 x}\, dx$$

$$= \int \left(1 + \frac{1}{\sin x} + \frac{\cos x}{\sin x} \cdot \frac{1}{\sin x}\right) dx$$

$$= \int (1 + \csc x + \cot x \cdot \csc x)\, dx$$

$$= x - \ln|\csc x + \cot x| - \csc x + C$$